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Simulation of the Electrical Parameters of Microarc Oxide Coatings Using the Matrix-Operator Method

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Abstract—The work addresses the issues of controllability of the microarc-oxidation process. This problem is mostly due to the difficulty of analytically characterizing the heterogeneous physical–chemical processes, which are nonlinear and nonstationary, that take place during microarc oxidation. To tackle this issue, a digital twin of the process is being developed, within which an analytical model is proposed to describe the behavior of the equivalent electrical circuit of the galvanic cell during coating deposition. The proposed analytical model of the process is nonlinear and nonstationary, which is attributed to the abrupt decrease in the active resistance of the coating during dielectric breakdowns. Based on experimentally obtained current and voltage oscillograms and the proposed analytical model, parametric identification of the electrical parameters of oxide coatings is performed using the matrix-operator method on the orthonormal Walsh basis. The matrix-operator method is selected because of its applicability to solving problems with both linear and nonlinear equations, as well as stationary and nonstationary parameters and variables, along with the relative simplicity of the mathematical and algorithmic implementation of computations. The calculations result in weight functions of the electrical model of the microarc-oxidation process, reflecting the relationship between the voltage and current in the galvanic cell (coating conductivity). The mean values of the weight functions (average conductivity) decrease during coating deposition, which confirms the adequacy of the proposed model and enables its use for implementing real-time procedures for monitoring coating properties. The scientific novelty of the work lies in the possibility of the real-time control of microarc oxidation through parametric identification of the electrical parameters of coatings using the matrix-operator method.

Keywords: microarc oxidation, electrical equivalent circuit, matrix operator method, parametric identification, Walsh functions, electrical parameters of coatings

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INTRODUCTION

The development of intelligent manufacturing technologies is one of the priority directions for digitalization of the Russian economy. Technologies aimed at creating new materials, particularly microarc oxidation (MAO), are of particular importance. Microarc oxidation is an electrochemical process of surface modification (oxidation) of valve metals and their alloys in an electrolytic plasma to obtain oxide layers (coatings). Microarc oxidation is performed in an electrolyte solution, where under the action of electrical-current pulses, microarc discharges occur on the surface of the treated part immersed in the electrolyte, altering its surface properties. Depending on the mode, oxide coatings with high microhardness and wear resistance [1], corrosion resistance [2], chemical stability [3], thermal protection [4, 5], antibacterial properties [6], and other enhanced characteristics can be formed. The area of application of MAO coatings is quite extensive and includes industries such as automotive, aerospace, shipbuilding, missile and defense industries, electronics, and medicine [7, 8].

Today, the main issue contributing to the low controllability and energy efficiency of the MAO technology is insufficient understanding of the coating-deposition mechanism [9]. The process of formation and modification of oxide layers during MAO involves a combination of numerous physical and chemical phenomena (electrical processes, electrochemical and plasma-chemical reactions, phase transitions) [10–14]. Due to the heterogeneity, nonlinearity, and nonstationarity of the phenomena involved, the mathematical description of such a process encounters significant difficulties, which are practically insurmountable within the framework of classical methods based on the use of differential equations [15]. Therefore, we need to develop alternative methods for analytically describing this process. These methods, based on the measurement results of the technological parameters of coating deposition (current and volt-

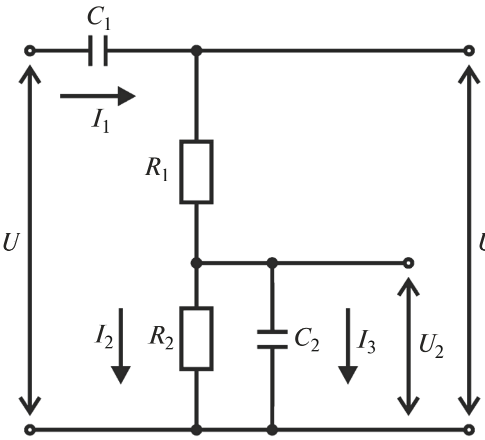

Fig. 1. Electrical equivalent circuit of the galvanic cell in the microarc-oxidation process.

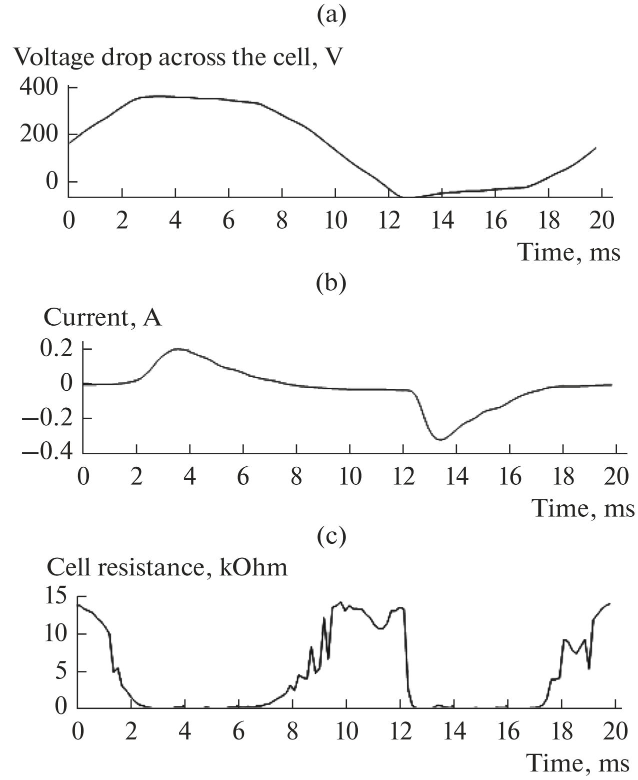

Fig. 2. Oscillograms of (a) voltage, (b) current, and (c) resistance of the galvanic cell.

age), would enable determination of the target coating parameters (thickness, porosity, microhardness, etc.).

In constructing nonlinear models of the MAO process, the use of the matrix-operator method appears promising. The characteristics of this method enable its application in solving problems with both linear and nonlinear equations, with stationary and nonstationary parameters and variables, ensuring high computational accuracy with relatively simple algorithmization.

In this study, parametric identification of the electrical parameters of the coatings is performed using the matrix-operator method. The developed mathematical model is integrated into a digital twin designed for intelligent control of the MAO process in real time.

EXPERIMENTAL

The formation of microarc-oxide coatings and measurement of the electrical parameters of the coating process were carried out using an automated software-hardware complex, the structure and operating principles of which are described in [16, 17]. Microarc-oxide coatings were applied to rectangular plates measuring 20 × 15 × 2 mm, made of aluminum alloy of AD31 grade. The samples were immersed in a galvanic bath with an electrolyte containing 0.5 g/L NaOH and 80 g/L Na_2SiO_3. Microarc oxidation was performed under sinusoidal current (current density of 10 A/dm^2, amplitude of the anode voltage of +400 V, anode-to-cathode current ratio of 1) for 30 min. During the MAO process, the current and voltage oscillograms were measured every minute. Mathematical processing of the obtained oscillograms was carried out using MATLAB v.R2020a [18].

NONLINEAR ELECTROPHYSICAL MODEL OF THE MICROARC-OXIDATION PROCESS

A nonlinear electrophysical model of the microarc-oxidation process was proposed in [19], within which an electrical equivalent circuit of the galvanic cell was developed (Fig. 1). The resistance of the electrolyte was modeled by an active resistance R_1, and the resistance of the component coating was represented by a parallel connection of a nonlinear active resistance R_2 and a reactive resistance of capacitance C_2. Capacitance C_1 is connected in series with the galvanic cell to limit the current.

The analysis of experimental oscillograms of the MAO process (Fig. 2) indicates the nonlinearity of the model, which is caused by the nonlinear dependence of the resistance of the coating of the part R_2 on the applied voltage (Fig. 3).

The symbolic form of equations describing the behavior of the electrical equivalent circuit of the MAO process (Fig. 2) is as follows:

$$\begin{cases} R_1 I_1(p) + R_2(p) I_2(p) = U_1(p) \\ R_2(p) C_2 p I_2(p) - I_3(p) = 0 \\ I_1(p) - I_2(p) - I_3(p) = 0, \end{cases} \tag{1}$$

where $p = j\omega$ is a complex argument, and ω is the frequency.

The equation of the relationship between the voltage applied to the cell U_1 and the current I_1 flowing through it, obtained as a result of solving system (1), is written as follows:

$$I_1(p) = \frac{U_1(T_2 p + 1)}{R_1 T_2 p + R_1 + R_2}, \quad (2)$$

where $T_2 = R_2 C_2$ is the time constant.

Let $U_1 = u$ and $I_1 = y$, then

$$y(t) = \int_0^t \omega(\tau) u(t - \tau) d\tau, \quad (3)$$

where $\omega(\tau)$ is the weight function of Eq. (2), and τ is time.

To calculate the unknown weight function $\omega(\tau)$, it is advisable to employ the method of matrix operators, widely used in the examination of nonstationary and nonlinear systems [20–23].

CALCULATION OF THE WEIGHT FUNCTION BY THE MATRIX-OPERATOR METHOD IN AN ORTHONORMAL WALSH BASIS

The matrix-operator framework is based on the theory of orthogonal functions [24, 25], which relies on employing Legendre, Chebyshev, and Laguerre polynomials, Walsh functions, and others. As a result, the time signals of the studied object are decomposed into orthogonal bases.

According to the method under consideration, the decomposition of signals $y(t)$, $u(t)$, and $\omega(\tau)$ of the studied object into orthogonal bases is represented as follows:

$$\begin{aligned} u(t) &= \mathbf{\Phi}^T(t)\mathbf{C}^u, \\ \omega(t) &= \mathbf{\Phi}^T(t)\mathbf{C}^\omega, \\ y(t) &= \mathbf{\Phi}^T(t)\mathbf{C}^y, \end{aligned} \quad (4)$$

here, $\mathbf{\Phi}^T(t)$ represents the system of basis functions, $\mathbf{C}^u$, $\mathbf{C}^y$, and $\mathbf{C}^\omega$ denote the matrix operators of the weighting coefficients. The equation for calculating the unknown matrix operator $\mathbf{C}^\omega$, obtained by substituting Eqs. (4) into Eq. (3), is expressed as follows:

$$\mathbf{\Phi}^T(t)\mathbf{C}^y = \int_0^T \mathbf{\Phi}^T(\tau)\mathbf{C}^\omega \mathbf{\Phi}(t - \tau)\mathbf{C}^u d\tau. \quad (5)$$

The matrix operator $\mathbf{C}^\omega$, found in the orthonormal Walsh basis, relates the voltage and current oscillograms of the galvanic cell, as depicted in Fig. 2. Selection of the orthonormal Walsh basis is justified by the ease of generating the basis, as Walsh discrete functions can be regarded as rows of the Hadamard matrix $\mathbf{H}$, the elements of which determine the value of functions at equidistant points, and by the simple procedure for computing the coefficients of expansion of the discrete function $X = x(kT_0)$ in terms of the Walsh functions:

$$C^x = n^{-1}\mathbf{H}X, \quad (6)$$

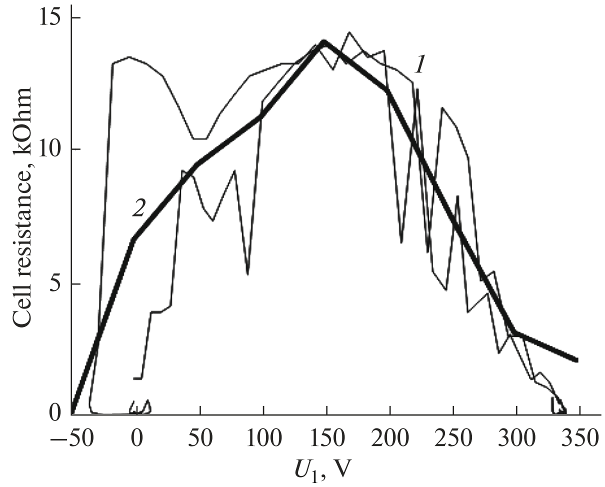

Fig. 3. (*1*) Experimental and (*2*) calculated dependences of the cell resistance on the applied voltage.

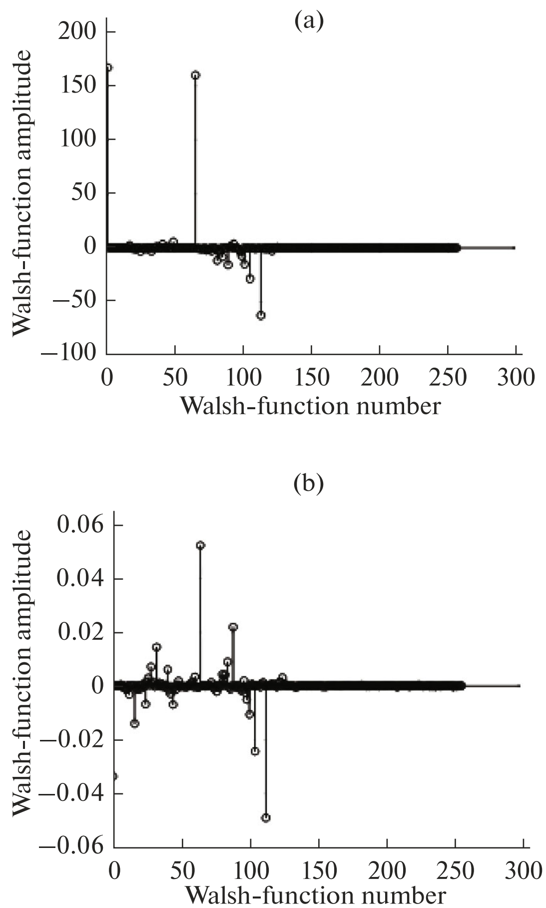

Fig. 4. Spectra of (a) voltage and (b) current in the Walsh basis.

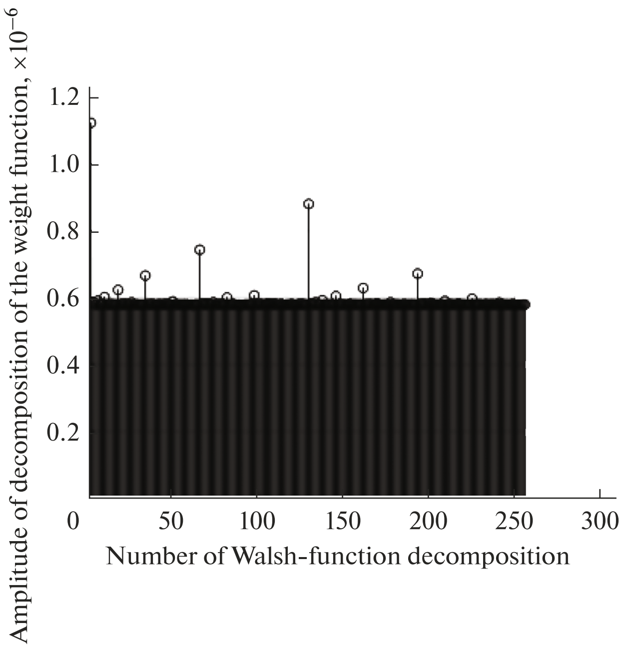

Fig. 5. Spectrum of the weight function in the Walsh basis.

where

$$\mathbf{H} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$$

is the 4 × 4 Hadamard matrix; $X = x(kT_0)$ is the discrete time function; $k = 0, 1, 2, 3, \ldots n$; and T_0 is the sampling period.

To calculate spectra on the Walsh basis, the realization duration needs to be set as $n = 2^k$. We select $k = 256$; then by Eq. (6), the spectra of voltage U_1 and current I_1 take the forms shown in Fig. 4.

The results of calculating the matrix operator of the weight function $\mathbf{C}^{\omega}$ according to Eq. (5) are presented in Fig. 5.

Based on the spectrum of the weight function, we calculated the desired weight function of the electrical equivalent circuit of the galvanic cell (Fig. 6), which reflects the change in the coating conductivity over time.

The negative conductivity value is due to the initial centering of the current and voltage signals. The actual change in the cell resistance is presented in Figs. 2 and 3.

We assessed the variation in the weight function of the electrical equivalent circuit of the galvanic cell during the MAO process. For this purpose, the oscillograms of two periods of currents and voltages were recorded every minute, and their corresponding weight functions were calculated. Figure 7 illustrates the three-dimensional representation of the changes in the matrix operators in the coordinates "experiment number–recording time."

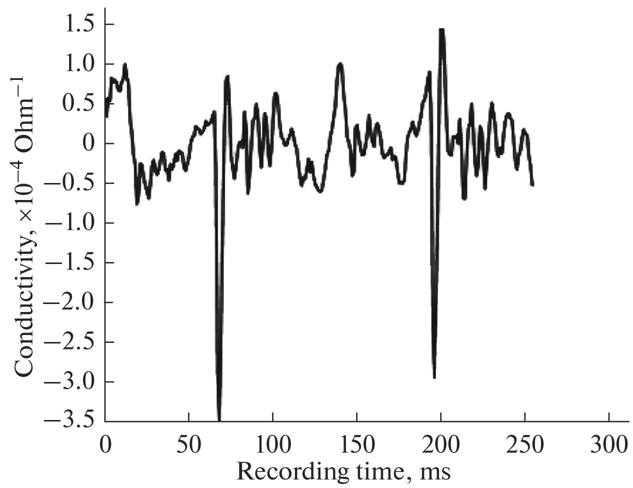

Fig. 6. Weight function of the electrical equivalent circuit of the galvanic cell.

Figure 8 shows the variation in the mean value of the weight function, indicating that during the MAO process, the conductivity of the galvanic cell initially increases sharply, then gradually decreases after several minutes. The alteration in conductivity or resistance of the cell enables one to infer about this resistance during the surface-modification process, and consequently, the properties of the applied coating.

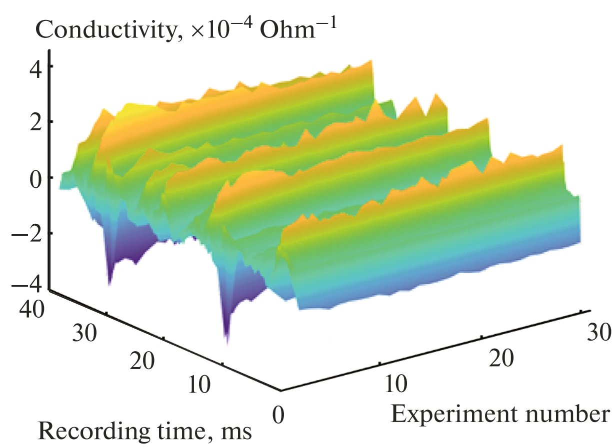

Fig. 7. Weight function of the electrical equivalent circuit of the galvanic cell in 30 experiments.

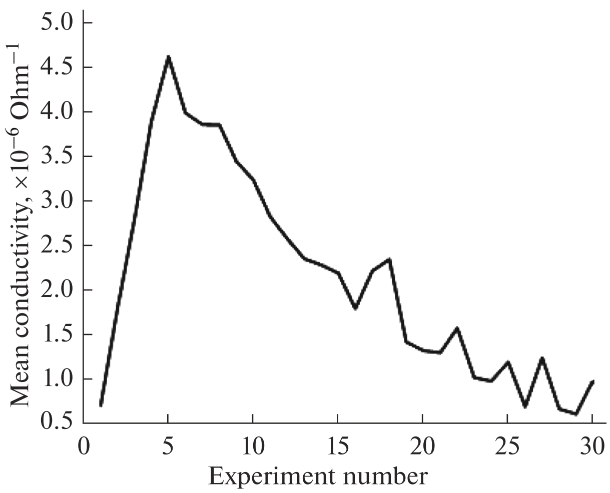

Fig. 8. Change in the mean value of the weight function (mean conductivity) during the MAO process.

CONCLUSIONS

The following key findings were obtained during the study.

Based on the experimentally obtained voltage and current oscillograms of the galvanic cell, we proposed an analytical model describing the behavior of the electrical equivalent circuit of the MAO process. Theoretical and experimental investigations revealed that the proposed analytical model of the MAO process is nonlinear and nonstationary.

We proposed to use the method of matrix operators to describe the nonlinear and nonstationary MAO process. The features of this method enable its application in solving problems with both linear and nonlinear equations, with stationary and nonstationary parameters and variables, ensuring high computational accuracy with relatively simple algorithms.

Using the method of matrix operators in the orthonormalized Walsh basis, the weight function of the galvanic cell was calculated, which has the physical meaning of the electrical conductivity of the coating.

During the MAO process, two periods of voltage and current of the galvanic cell were recorded every minute. The conductivity of the coating was calculated using the obtained oscillograms. The results of the calculations showed that during the MAO process, the conductivity of the coating initially sharply increases, and then gradually decreases after several minutes. The observed change in the conductivity of the coating allows for estimation of its physical-geometric parameters (thickness and porosity) during the process.

The proposed model can be used in the development of a digital twin of the microarc-oxidation process.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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