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#### **PAPER • OPEN ACCESS**

# Mirror symmetry in transverse oscillations of ropes and shafts in electromechanical systems

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## Mirror symmetry in transverse oscillations of ropes and shafts in electromechanical systems

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**Abstract:** The article explores the possibility of developing a mathematical description of the bending of a round beam, based on end supports, to construct a theory of oscillations of a curved shaft of a mine hoist. The results of the study allow to develop a model of transverse oscillations of the elastic shaft and a long rope in order to synthesize a microcontroller control system.

#### 1. Introduction

The transverse bending of the shaft is closest to that of the round beam (figure 1). But the beam rests freely on the supports, its ends bending upwards. Unlike the beam, the shaft is fixed in bearings, connected to other devices that prevent bending, which excludes the possibility to use theory [1, 2, 3, 4, 5] for the shaft.



**Figure 1.** Experimental bending of a thin elastic round shaft.

The rope is pulled by a load and approaches the string in flexibility. However, according to Figure 2, the string is broken at the points of attachment and retraction, which is physically impossible for the rope. At the same time, the attachment of the rope ends is closer to the attachment of the shaft ends. So fastening the rope to the vessel with the load also does not allow a kink, and the rope runs onto the hoist drum and the pulley of the headframe tangentially to the circle, which physically excludes bending at the end point of tangency of the pulley circumference. It follows that the rope and shaft are subject to the same theory. However, the additional accounting of rope tension will increase the accuracy of the mathematical model.



Figure 2. Bend string and mirror line 1 to line 2.

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#### 2. Results and discussion

Figure 3 shows the rope of the physical model of the mine hoist. At the pull point, the rope bends without a break, just like the shaft, but unlike a shaft with a smaller radius of curvature.



Figure 3. Rope, hoist drum and headframe of the laboratory stand of mine hoist.

The theory of bending in [4, 5] is built using bending moments M (Figure 4) applied to the beam ends, which made it possible to solve the problem of bending a freely supported beam, but it is impossible to fulfill the condition of pinching the shaft ends.



**Figure 4.** To the theory of bending of the beam freely supported at the ends.

The solution of this problem with respect to the shaft is based on Figure 5, which shows the concentrated force *P*, which is applied to the right face x = a of the selected element of infinitely small length *dx*. The view of the element before and after bending is shown in Figure 5.



**Figure 5.** Bending of rope and shaft due to concentrated force.

Applying force *P* causes the shaft and rope to bend. When bent, the length of the axis  $O_1$ - $O_2$  (Figure 4) of the shaft and rope remains unchanged, and the fibres are compressed above the axis and stretched below. Compressive and stretching forces according to Hooke's law grow when bending increases, compensating for the bending action of force *P*, the bending process will stop only when the sum of compressive and stretching forces is equal to force *P*.

From Figure 5 we have dM(x) = Pdx and integrating we get the moment-of-force function P  $M(x) = Px + C_1$ . But force P creates not one, but two moments with shoulders a and " $L_b - a$ , and the distribution of force P for two moments is determined by the reaction of supports  $P_1$  and  $P_2$  (Figure 6), which are inversely proportional to the lengths of shoulders [1, 3, 8], in the consequence of which for the left support we have

from where



Figure 6. Design diagram of curved round beam.

By substituting  $P_1$  in the moment function, we get

$$M(x) = \frac{P(L_b - a)}{L_b} x + C_l, \quad 0 \le x \le a.$$

$$\tag{1}$$

The integration in Figure 6 extends only to the left side of the shaft, the solution of the problem with respect to the right side remains open. To solve the problem, we will use the same modes of fixing the shaft ends and ideas of mirror symmetry. The demonstration of the idea will be performed on a simple experiment with a string fixed on two supports A and B. Figure 2 shows the string in a pulled-off state and its mathematical description, borrowed from [1] has the form

$$h_1(x) = h_{ot} \cdot \frac{x}{a}, \qquad 0 \le x \le a$$
 (2,a)

$$h_2(x) = h_{ot} \cdot \frac{L_b \cdot x}{L_b \cdot a}, \qquad a \le x \le L_b$$
(2,b)

In Figure 2, we introduce the second coordinate system, the origin  $x^* = 0^*$  of which is located on the support B, and the x-axis  $x^*$  is directed from right to left, opposite the x-axis (we change the sign to the opposite) with the origin on the support 0. Based on the similarity of triangles we write

$$\frac{h_1(x)}{h_{ot}} = \frac{x}{a}, \qquad \frac{h_2(x)}{h_{ot}} = \frac{x^*}{a^*}, \qquad \text{откуда}$$
$$h_1(x) = h_{ot}\frac{x}{a}, \qquad 0 \le x \le a, \qquad (3,a)$$

$$h_2(x^*) = h_{ot} \frac{x^*}{a^*}, \quad a \le x \le L_b.$$

$$(3,b)$$

Figure 2 plots the functions (3, a) line 1 and the functions (3, b) - line 2. The plots are constructed in different coordinate systems and illustrate mirror symmetry, confirming the same operation modes of the supports. To construct the bending functions, the function (3, b) from the coordinate system  $x^*$ must be transformed into the system x.

According to [7] subject to conversion

$$x^* = L_b - x, \qquad a^* = L_b - a \tag{4}$$

as a result (3) will take the final form

$$h_1(x) = h_{ot} \cdot \frac{x}{a}, \qquad 0 \le x \le a, \tag{5,a}$$

$$h_2(x) = h_{ot} \cdot \frac{L_b \cdot x}{L_b \cdot a}, \quad a \le x \le L_b,$$
(5,b)

which corresponds to the graph of lines 1 and 3 in figure 2.

Using mirror symmetry, we write the bending function in the coordinate system  $x^*$ 

$$M(x^{*}) = \frac{P(L_{b} - a^{*})}{L_{b}} x^{*} + C_{2}, \qquad a \le x \le L_{b}.$$
(5,c)

Applying the transformation (4) to (5, c), we obtain

$$M(x) = \frac{P \cdot a}{L_b} (L_b \cdot x) + C_2, \qquad a \le x \le L_b$$
(5,d)

Thus, for (1) and (5, d), we have

$$\mathbf{M}(\mathbf{x}) = \begin{cases} \frac{P(L_b \cdot a)}{L_b} \mathbf{x} + C_1, & 0 \le \mathbf{x} \le a, \\ -\frac{P \cdot a}{L_b} \mathbf{x} + P \cdot a + C_2, & a \le \mathbf{x} \le L_b. \end{cases}$$
(6)

In sources [4, 5], only bending moments are taken into account, taking into account the additional dynamic moment and the moment of internal viscous friction  $M_t(t)$ , we obtain

$$EJ\frac{d^{2}h(x,t)}{dx^{2}} = M(x) + M(t) - M_{c}(t) - M_{t}(t),$$
(7)

where according to the source [8] we have

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$$\mathbf{M}(\mathbf{t}) = \begin{cases} (L_b - a)m \frac{d^2 h(t)}{dt^2}, & 0 \le x \le a, \\ am \frac{d^2 h(t)}{dt^2}, & a \le x \le L_b. \end{cases}$$
(8)

$$\binom{dm}{dt^2} \frac{dt^2}{dt}, \qquad dt \leq x \leq L_b.$$

$$M_t(t) = C_t L_b \eta D \frac{dh(t)}{dt}, \qquad C_t L_b = c_t$$
(9)

By substituting (6), (8) and (9) in (7), we obtain a differential equation in the second-order partial derivatives describing the damping oscillations of the shaft and rope

$$\mathrm{EJ}\frac{\partial^{2}\mathbf{h}(\mathbf{x},\mathbf{t})}{\partial \mathbf{x}^{2}} - \mathrm{Am}\frac{\partial^{2}\mathbf{h}_{1}(\mathbf{x},\mathbf{t})}{\partial \mathbf{t}^{2}} - \mathrm{B}\frac{\partial\mathbf{h}_{1}(\mathbf{x},\mathbf{t})}{\partial \mathbf{t}} = \begin{cases} \frac{P(L_{b}-a)}{L_{b}}\mathbf{x} + C_{l}, \ A = L_{b}-a \ 0 \leq \mathbf{x} \leq a, \\ -\frac{P \cdot a}{L_{b}}\mathbf{x} + Pa + C_{2}, \ A = a, \ a \leq \mathbf{x} \leq L_{b}. \end{cases}$$
(10)

Constant integration is determined from steady-state mode, when

.

$$\frac{\partial^2 h(x,t)}{dt^2} = \frac{\partial h(x,t)}{dt} = 0,$$

in this case, the partial differential equation (10) is transformed into an ordinary differential equation

$$\mathrm{EJ}\frac{\mathrm{d}^{2}\mathbf{h}(\mathbf{x},\mathbf{t})}{\mathrm{dx}^{2}} = \begin{cases} \frac{P(L_{b}-a)}{L_{b}}\mathbf{x}+C_{l}, & 0 \leq \mathbf{x} \leq a, \\ \frac{Pa}{L_{b}}(L_{b}-\mathbf{x})+C_{2}, & a \leq \mathbf{x} \leq L_{b}. \end{cases}$$
(11)

Integrating twice, we get

$$\mathrm{EJ}\frac{\mathrm{dh}(\mathbf{x},\mathbf{t})}{\mathrm{dx}} = \begin{cases} \frac{P(L_{b}-a)}{2L_{b}}x^{2} + C_{I}x + C_{3}, & 0 \leq x \leq a, \\ -\frac{Pa}{2L_{b}}\left(L_{b}-x\right)^{2} - C_{2}\left(L_{b}-x\right) + C_{4}, & a \leq x \leq L_{b}. \end{cases}$$
$$\mathrm{EJh}(\mathbf{x}) = \begin{cases} \frac{P(L_{b}-a)}{6L_{b}}x^{3} + C_{I}\frac{x^{2}}{2} + C_{3}x + C_{5}, & 0 \leq x \leq a, \\ \frac{P\cdot a}{6L_{b}}\left(L_{b}-x\right)^{3} + C_{2}\frac{\left(L_{b}-x\right)^{2}}{2} - C_{4}\left(L_{b}-x\right) + C_{6}, & a \leq x \leq L_{b}. \end{cases}$$

Fulfilling the conditions for fixing the ends of the shaft in the bearings, we obtain the boundary conditions

$$h(0) = h'(0) = h(L_b) = h'(L_b) = 0$$
 (12)

we get  $C_3 = C_4 = C_5 = C_6 = 0$ , the substitution of which gives

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$$EJ\frac{dh(x,t)}{dx} = \begin{cases} \frac{P(L_b - a)}{2L_b} x^2 + C_1 x, & 0 \le x \le a, \\ -\frac{P \cdot a}{2L_b} (L_b - x)^2 - C_2 (L_b - x). & a \le x \le L_b. \end{cases}$$
(13)

$$\mathrm{EJh}(\mathbf{x}) = \begin{cases} \frac{P(L_b - a)}{6L_b} x^3 + C_1 \frac{x^2}{2}, & 0 \le \mathbf{x} \le a, \\ \frac{P \cdot a}{6L_b} (L_b - x)^3 + C_2 \frac{(L_b - x)^2}{2}, & a \le \mathbf{x} \le L_b. \end{cases}$$
(14)

The integration constants  $C_1$  and  $C_2$  are determined from the condition of continuity and smoothness of the function. For this, the first function from (14) must be equated with the second and the first derivative from (13) and the second with x = a. Then, grouping the terms in the obtained equalities, we have

$$C_{1}a+C_{2}(L_{b}-a)=-\frac{Pa^{2}(L_{b}-a)}{2L_{b}}-\frac{P\cdot a(L_{b}-a)^{2}}{2L_{b}},$$
  
$$C_{1}\frac{a^{2}}{2}-C_{2}\frac{(L_{b}-a)^{2}}{2}=\frac{P\cdot a(L_{b}-a)^{3}}{6L_{b}}-\frac{Pa^{3}(L_{b}-a)}{6L_{b}}.$$

We multiply the first equation by  $\frac{L_b - a}{2}$ , add it to the second, and solve the result with respect to  $C_1$ 

$$C_1 = \frac{P(a^2 - L_b^2)}{6L_b}.$$
 (15,a)

The first equation is multiplied by  $\frac{a}{2}$  and subtract the second from it, resolving with respect to  $C_2$ , we obtain

$$C_{2} = \frac{P \cdot a \left(3 a L_{b} - a^{2} - 2 a L_{b}^{2}\right)}{6 \left(L_{b}^{2} - a L_{b}\right)}$$
(15,b)

#### 3. Conclusion

The simulation result is shown in Figure 7. The curves were recorded using the support and research system developed by the author. Figure 7 a) shows the bending of the shaft from the action of the force P applied to the middle of the shaft  $a = \frac{L_b}{2}$ , n figures 7 b) and 7 c) the force P was applied to the points  $a = \frac{L_b}{4}$  and  $a = \frac{3L_b}{4}$ .



Figure 8 shows the experimental curves, and figure 8 a) corresponds to figure 7 a), figure 8 b) to figure 7 b), and figure 8 c) to figure 7 c). A comparison of the figures shows the coincidence of the characteristics of theoretical processes with experimental ones.



**Figure 8.** Experimentally obtained bending of the shaft from the force P acting at the point  $a = \frac{L_b}{2}$  for a), at the point  $a = \frac{L_b}{4}$  for b) and at the point  $a = \frac{3L_b}{4}$  for c).

Thus, based on mirror symmetry using two coordinate systems, a universal mathematical model of the transverse oscillations of the shaft and the rope from the action of a concentrated load is created. The model describes forced and free oscillations, as well as a steady state. Accounting for internal viscous friction provides an investigation of damped oscillations.

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