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# Biphase Model of Plastic Deformation in Electric Fields

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**Abstract.** The object of the research is creep deformation proceeding in the conditions of electrostatic field effect. The purpose of the research is to develop the mathematical model of creep under the electrostatic field effect from the positions of representations about the wave nature of plastic deformation process. The theoretical studies of electrostatic field effect being characterized by small (up to ± 1V) potentials on the basis of mass, momentum and energy conservation in two-dimensional formulation were carried out in the process of research. The material being deformed was represented as two phase heterogeneous medium. The first component is excited and being responsible for structure transformation, the second one is unexcited and disconnected with them. For each of the components the laws of mass and momentum conservation were written. For electric fields the Maxwell equations were written. For the first time the two phase filtration model of creep was developed as a result of the research. The model takes into account the inhomogeneity of plastic deformation under electrostatic field effect. The dispersion relation for the waves of plasticity is obtained.

## Introduction

The establishing of physical mechanisms of plastic deformation in the condition of electric effects has long called the attention of researchers. Nowadays the results indicating that electric state of surface influence the metals' resistance to creep deformation [1 ,2], microindentation [3] and stress relaxation [4] are obtained. One of the reasons for mechanical characteristics' change of materials is the change in their surface tensions which, as is known, has effect on the dislocation subsystem [5]. The other reason (according to the authors' opinion [6]) is the presence of double electric layer leading to the abrupt increase in energy of the system «conductor – double electric layer», and the electric field localized in this layer decreases the value of surface barrier separating the elastically strained state from the non-elastically strained one. Creep increases as a result of rate. The study of crystal lattice stability in electric field by the molecular dynamics method [7] showed that when electron density decreased the increase in distance between ions took place. It, as evidenced by the results of the research [8], may be the reason for metals' hardening. As the process of plastic deformation has a wave nature then it should be expected that electrostatic effect influences the characteristics of plasticity wave. In the research [9] it is established that electrostatic field with potential 1V increases by two times in the rate of motion of localization sites in the conditions of commercially pure aluminum creep but the wave length remains constant. However, the mechanism of this increase remain still unclarified to completion although the studies [10] indicate that the change in surface energy contribute to the formation of large scale sites of localization. Thus, it should be concluded that the reason for the change in creep rate under the electric field effect is restructuring of metal surface layers leading to the change in distance between ions in electron subsystem. As the results of study [10] shows the elastic component in total deformation plays the important part in plastic deformation development and it should not be neglected. Therefore, it is necessary to divide the material into two phases for the development of plastic flow model in electric field. Note that biphase approach is used successively in modeling of gas suspensions, water saturated soils, oil stratums [11, 12].

**Problem Formulation**

Represent the material being deformed as a biphase heterogeneous mixture. The similar representations were used in the theory of superconductivity and superfluidity [13]. The dispersion relation established in [10, 14] is identical to the dispersion law for superfluid helium indicative of the presence of two motions. Choose the elastic motion of lattice ions («fluid») as the excited component and their plastic motion («skeleton») as the unexcited one. Write the laws of momentum and mass conservation for each phase with regard to the external action [11, 12]. They will take the form

$$\begin{aligned}
&\rho_1 \frac{d_1 \vec{u}_1}{dt} = \alpha \operatorname{div} \tilde{\sigma} + \varphi(\vec{u}_2 - \vec{u}_1) + F_1 \\
&\frac{d_1 \rho_1}{dt} + \rho_1 \operatorname{div} \vec{u}_1 = 0 \\
&\rho_2 \frac{d_2 \vec{u}_2}{dt} = (1-\alpha) \operatorname{div} \tilde{\sigma} - \varphi(\vec{u}_2 - \vec{u}_1) + F_2 \\
&\frac{d_2 \rho_2}{dt} + \rho_2 \operatorname{div} \vec{u}_2 = 0
\end{aligned} \tag{1}$$

where $F_i = \sigma \vec{\nabla} \alpha + \varphi(\vec{u}_2 - \vec{u}_1) + \rho_i \beta \vec{E}$, $\rho_1 = \alpha \rho_e$, $\rho_2 = (1-\alpha)\rho_s$, $\tilde{\sigma}_1 = \alpha \tilde{\sigma}$, $\tilde{\sigma}_2 = (1-\alpha)\tilde{\sigma}$, where $\rho_1$, $\rho_2$, $\rho_e$, $\rho_s$ – the reduced and true densities of the first and second phases respectively, $\alpha$ – volume fraction of the first phase, $\tilde{\sigma}$ – stress in the whole mixture, $u_1$ and $u_2$ – rates of the first and second phase respectively, coefficient depending on the volume fraction of the first phase; E – electron field strength; $\beta$ – a constant. We consider that electric field acts on the elastic component only. According to the research [15], neglect the inertial term in the first equation of the system Eq. 1. We shall get as a result:

$$\alpha \operatorname{div} \tilde{\sigma} = -\varphi(\vec{u}_2 - \vec{u}_1) - \rho_1 \beta \vec{E} \tag{2}$$

For closure of system Eq. 1 it is necessary to write the equation of state, Maxwell equation and relation between stress and pressure:

$$P = A_1 \frac{\rho_1}{\alpha}, \rho_s = const, div\vec{E} = \rho, \tilde{\sigma} = -P \tag{3}$$

where $\rho$ – density of charges on the surface the sample. We shall suppose it to be constant. Pass on to the consideration of the problem in two-dimensional formulation. In view of Eq. 2 and Eq. 3 the system Eq. 1 will take the form:

$$\begin{aligned}
&\frac{\partial \alpha}{\partial t} + u_{2x} \frac{\partial \alpha}{\partial x} + u_{2y} \frac{\partial \alpha}{\partial y} = (1-\alpha)\left( \frac{\partial u_{2x}}{\partial x} + \frac{\partial u_{2y}}{\partial y} \right) \\
&\frac{\partial u_{2x}}{\partial t} + u_{2x} \frac{\partial u_{2x}}{\partial x} + u_{2y} \frac{\partial u_{2x}}{\partial y} = -\frac{1}{(1-\alpha)\rho_s} \frac{\partial P}{\partial x} + \frac{\beta \rho_1 E_x}{(1-\alpha)\rho_s}; \\
&\frac{\partial u_{2y}}{\partial t} + u_{2x} \frac{\partial u_{2y}}{\partial x} + u_{2y} \frac{\partial u_{2y}}{\partial y} = -\frac{1}{(1-\alpha)\rho_s} \frac{\partial P}{\partial y} + \frac{\beta \rho_1 E_y}{(1-\alpha)\rho_s} \\
&\frac{\partial \rho_1}{\partial t} + \frac{\partial (\rho_1 u_{1x})}{\partial x} + \frac{\partial (\rho_1 u_{1y})}{\partial y} = 0 \\
&u_{1x} = u_{2x} - \frac{\alpha}{\varphi} \frac{\partial P}{\partial x} + \frac{\beta}{\varphi} \rho_1 E_x; u_{1y} = u_{2y} - \frac{\alpha}{\varphi} \frac{\partial P}{\partial y} + \frac{\beta}{\varphi} \rho_1 E_y.
\end{aligned} \tag{4}$$

It follows from Maxwell equation that field stress being created by the charged conductor as a plane takes the form:

$$E = \frac{\rho}{2\varepsilon_0} \text{ or } E = \frac{C\varphi_0}{2\varepsilon_0 ab} \tag{5}$$

where C – capacity of conductor, $\varphi_0$ – potential on the surface of conductor, a and b – sizes of the plate, $\varepsilon_0$ – electric constant. We defined the coefficient $\varphi$ as in [11]:

$$\varphi = \frac{K(1-\alpha)\rho_1}{\alpha}, \tag{6}$$

Linearize the system Eq. 4 about small disturbances: $\alpha = \alpha_0 + h\tilde{\alpha}$, $u_{2x} = u_0 + h\tilde{u}$, $u_{2y} = w_0 + h\tilde{w}$, $\rho_1 = \rho_0 + h\tilde{\rho}$, where h – small parameter. As a result we obtain:

$$\begin{aligned}
&\frac{\partial\tilde{\alpha}}{\partial t} + u_0\frac{\partial\tilde{\alpha}}{\partial x} + w_0\frac{\partial\tilde{\alpha}}{\partial y} = (1-\alpha_0)\left(\frac{\partial\tilde{u}}{\partial x} + \frac{\partial\tilde{w}}{\partial y}\right)\\
&\frac{\partial\tilde{u}}{\partial t} + u_0\frac{\partial\tilde{u}}{\partial x} + w_0\frac{\partial\tilde{u}}{\partial y} = -\frac{A_1}{(1-\alpha_0)\alpha_0\rho_s}\left(\frac{\partial\tilde{\rho}}{\partial x} - \frac{\partial\tilde{\alpha}}{\partial x}\frac{\rho_0}{\alpha_0}\right)\\
&\frac{\partial\tilde{w}}{\partial t} + u_0\frac{\partial\tilde{w}}{\partial x} + w_0\frac{\partial\tilde{w}}{\partial y} = -\frac{A_1}{(1-\alpha_0)\alpha_0\rho_s}\left(\frac{\partial\tilde{\rho}}{\partial y} - \frac{\partial\tilde{\alpha}}{\partial y}\frac{\rho_0}{\alpha_0}\right) + \frac{\beta E_y(\tilde{\rho}(1-\alpha_0) - \rho_0\tilde{\alpha})}{(1-\alpha_0)^2\rho_s}\\
&\frac{\partial\tilde{\rho}}{\partial t} + \left(u_0 + \frac{\beta\alpha_0 E_y}{K(1-\alpha_0)}\right)\frac{\partial\tilde{\rho}}{\partial x} + (w_0 + \frac{\beta\alpha_0 E_y}{K(1-\alpha_0)})\frac{\partial\tilde{\rho}}{\partial y} + \rho_0\left(\frac{\partial\tilde{u}}{\partial x} + \frac{\partial\tilde{w}}{\partial y}\right) =\\
&= \frac{\alpha_0}{K(1-\alpha_0)}\left(\frac{\partial^2\tilde{\rho}}{\partial x^2} + \frac{\partial^2\tilde{\rho}}{\partial y^2} - \frac{\rho_0}{\alpha_0}\left(\frac{\partial^2\tilde{\alpha}}{\partial x^2} + \frac{\partial^2\tilde{\alpha}}{\partial y^2}\right)\right) + \frac{\beta\rho_0 E_y}{K(1-\alpha_0)^2}\frac{\partial\tilde{\alpha}}{\partial y}
\end{aligned} \tag{7}$$

## Results and Discussion

Represent the solution of the system Eq. 7 in the form of running wave:

$$\begin{aligned}
\tilde{\alpha} &= A\exp(-i(\omega t - k_x x - k_y y))\\
\tilde{u} &= B\exp(-i(\omega t - k_x x - k_y y))\\
\tilde{w} &= C\exp(-i(\omega t - k_x x - k_y y))\\
\tilde{\rho} &= F\exp(-i(\omega t - k_x x - k_y y))
\end{aligned} \tag{8}$$

As a result we shall obtain the system of linear algebraic equation whose determinant will permit to find the dispersion equation for the running waves. In the absence of electric fields it will take the form:

$$\begin{aligned}
&((((-1+\alpha_0)\rho_s\alpha_0^2 K)\omega^2 + \rho_s\alpha_0^2 A_1\omega(2K(u_0k_x + w_0k_y)(1-\alpha_0) + i(k_x^2 + k_y^2)\alpha_0)\\
&+\rho_s\alpha_0^2 A_1(u_0k_x + w_0k_y)\Big(K(u_0k_x + w_0k_y)i(k_x^2 + k_y^2)\Big)))\Big(-i\omega + u_0k_x + w_0k_y\Big)^2 = 0
\end{aligned} \tag{9}$$

The transformation of $\omega = \Omega + u_0k_x + w_0k_y$ reduces Eq. 9 to the following form:

$$((\alpha_0 - 1)\rho_s^2\alpha_0^2 K\Omega^2 - i\Omega\rho_s A_1\alpha_0^3 k^2 + \rho_0 A_1 K k^2)\Omega^2 = 0 \tag{10}$$

where $k^2 = k_x^2 + k_y^2$. The expression in Eq. 10 being in the parenthesis coinside in the form with the dispersion equation obtained in [17] for the waves of plasticity. Its roots take the form:

$$\Omega_{1,2} = 0$$

$$\Omega_{3,4} = \frac{i\alpha_0 A_1 k^2}{2(\alpha_0 - 1)\rho_s K} \pm \frac{\sqrt{\left(4A_1\rho_0(1-\alpha_0)K^2 - A_1^2\alpha_0^4 k^2\right)}k}{2\rho_s\alpha_0(\alpha_0 - 1)K} \tag{11}$$

Deduce the critical value of wave number and wave length at which the instability of stationary state of relatively small disturbances occurs. For it the condition $Im(\Omega) > 0$ should be met which is achieved at the negative discriminant of the equation Eq. 11. It is achieved at wave number values being equal to

$$k > \frac{2K}{A_1\alpha_0^2}\left(A_1\rho_0(1-\alpha_0)\right)^{1/2} \tag{12}$$

then the critical wavelength will take the form:

$$\lambda_* = \frac{2\pi}{k_*} = \frac{\pi A_1\alpha_0^2}{K}\left(\frac{1}{A_1\rho_0(1-\alpha_0)}\right)^{1/2} \tag{13}$$

where $K = \frac{A_1\rho_0}{\eta_1}$. At values $\lambda < \lambda_*$ the small disturbances increase. It means that the presented model may describe the spontaneous decay of shear band into fragments. The sizes of fragments are proportional to the critical wavelength. Its estimate at $\alpha_0 = 0.1$, $\rho_0 = 7\,800$ kg/m$^3$, $\eta_1 = 10^5$ Pa·s, $A_1 = 10^7$ m$^2$ / s$^2$ gives the value $\lambda_* \sim 1.41$ μm. The sizes of «white layer» and deformation localization region found in [16] have the same order.

In electrostatic fields the dispersion equation Eq. 10 is added by the terms containing the electric field strength

$$\Omega\left(\Omega^3 + \Omega^2\left(\frac{A_1k^2 i - \alpha_0\beta kE}{(1-\alpha_0)K}\right) + \Omega\left(\frac{A_1k^2\alpha_0^2\rho_0 - i\rho_0\beta kE}{(1-\alpha_0)\alpha_0^2\rho_s}\right) - \left(\frac{ik^2\rho_0\beta^2E^2}{(1-\alpha_0)\rho_s K} + \frac{A_1k^3\rho_0\beta E}{(1-\alpha_0)\alpha_0\rho_s K}\right)\right) = 0 \tag{14}$$

The analytical form of roots of Eq. 14 is cumbersome therefore they will not be extracted. Restrict ourselves by the numerical solution of Eq. 14 at $\alpha_0 = 0.1$, $\rho_0 = \rho_s = 7800$ kg/m$^3$, $\eta_1 = 10^5$ Pa·s, $A_1 = 10^7$ m$^2$/s$^2$, $E \approx 10^9$ V/m, $\beta \approx 10^{-4}$ C/kg. The numerical calculations showed that the most unstable modes were those with wave number $k > 10^7$ m$^{-1}$. It means that the electric field application displaces the instability of the material plastic flow into the range of submicro and nanodimensional wavelengths.

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