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Identification Problems of Material Structures Based on Fractal Presentations

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Abstract—This work is devoted to management of material structures based on fractal presentations. Fractal material structures are formed due to positive feedback. The first stage is described: formulation of the identification problem of material structures based on fractal presentations. Fractal nature of materials is mentioned in numerous recent studies, herewith, generation of fractal structures is attributed to mechanisms of positive feedbacks. Fundamental physicochemical regularities of origination and transformation of material structures are currently developed and presented in such form that it is difficult to apply them for synthesis of control algorithms of structures. In other words, they do not meet the requirements to control models: output impacts are not disclosed as a function of external factors. It would be reasonable to develop fractal models of structures (that is, to identify material structures) with subsequent development of control impacts, in particular on parameters of positive feedback, in order to predict and modify material structure as required. This corresponds to the synthesis of control algorithms with estimation of controlled object and selection of coefficients of controller gain. The identification problems of images of actual material structures are formulated on the basis of presentations of dynamic chaos. These formulations of the problems have been used for development of methods and algorithms of identification of material structures in various industries, including mining and metallurgy.

Keywords: material structure, positive feedback, objects with positive feedback, identification, fractal models

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INTRODUCTION

Conventional geometry methods, which are widely applied in natural sciences upon description of material structures, including materials science, mining, oil and gas industry, and mechanics of deformable bodies, are based on approximation of analyzed body structure by geometrical figures, for which metric and topological dimensions are equal to each other. Herewith, the internal structure of the analyzed object is ignored. This results in losses of a significant portion of data regarding properties and behavior of the analyzed systems, which are substituted by more or less adequate models [1–3].

However, there are situations when topologically non-equivalent models cannot be applied in principle. The fractional metric dimension of such objects not only characterizes their geometric image but also reflects their origination and evolution, as well as determines their dynamic properties [4–7].

Presently, numerous real and model mechanical, physical, chemical, and biological systems are known, in which determinate chaos exists [8–18]. Despite the fact that mathematics deals with geometric objects of fractional dimension, the notions of fractals only recently became the basis for consideration of surrounding natural forms, including description of material structures [19–23].

The processes, producing fractal structures of materials, are common processes with positive feedback, in which one and the same operation is performed again and again, and the result of one iteration is the initial value for another iteration (see Fig. 1) [5].

The dependence between result and initial value, that is, the dynamic law $X_{n+1} = f(X_n)$, is expressed as follows [10]:

$$X_{n+1} = f(X_n) = X_n^2 + c;$$

$$X_{n+1} = f(X_n) = (1 + r)X_n - rX_n^2,$$

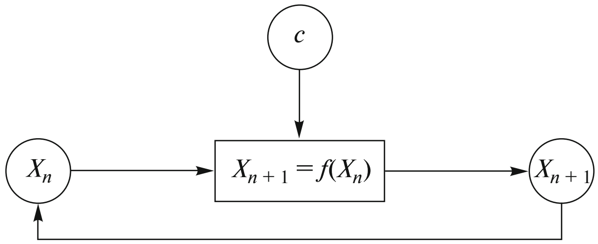

Fig. 1. Schematic view of formation of fractal structure.

where X_n is the variable; $f(X_n)$ is the transformation function; c is the parameter, complex variable, whereas X_n, $c \in C$; r is the coefficient of increment.

This work is aimed at figuring out the formulation of the problems regarding the identification of material structures based on fractal presentations.

RESULTS AND DISCUSSION

During the studies, the problems of image identification of actual material structures were formulated [24, 25]:

— by means of fractal models based on known (typical) fractals;

— by means of fractal models generated by the formation mechanism of fractal structure.

Experimental methods of fractal analysis assume the approximation of real (actual) structures of materials by fractals of a certain generation. The images of actual structures can be described (identified) by two approaches. The first approach is to select setting coefficients or parameters upon known mechanism of structure formation. This approach is complicated by the fact that fundamental studies devoted to such formation mechanisms are unavailable. The second approach is the description of actual structures using exactly the fractal presentations.

With the identification of materials using the second approach, two variants are possible:

— structure description by previously prepared fractals;

— generation of fractal structure using known algorithms.

IDENTIFICATION BY FRACTAL MODELS BASED ON KNOWN (TYPICAL) FRACTALS

In the first variant of identification by means of ready fractals, depending on the final target of the model development of material structure and available a priori information, it is possible to develop three formulations of the problems.

Formulation of Problem I

It is given:

1. Actual structures of materials

$$St_j; \quad j = \overline{1, J},$$

where j is the structure number; J is the amount of structures.

2. Set of typical fractals

$$Fr_l; \quad l = \overline{1, L},$$

where l is the number of typical fractal; L is the amount of fractals.

3. Identification criterion

$$Q_{j,l} = \sum_{m;n;l=1}^{M,N,L} \left| St_j(m,n) - Fr_l(m,n) \right| \to \min,$$

where m, n is the amount of points on structure image.

4. Constraint

$$M \in \overline{1, M^{\max}}; \quad N \in \overline{1, N^{\max}},$$

where $M^{\max}$, $N^{\max}$ is the maximum amount of points.

It is required to select such fractal from the set Fr_l, for which Q is minimum.

Formulation of Problem II

It is given:

1. Actual structures of materials

$$St_j; \quad j = \overline{1, J}.$$

2. Set of typical fractals

$$Fr_l; \quad l = \overline{1, L}.$$

3. Model of material structure

$$St_j^M = \sum_{j;l=1}^{J,L} \alpha_{ij} Fr_l + \varepsilon_j; \quad j = \overline{1, J},$$

where α_{lj} are the target weight coefficient; ε_j is the model error (disturbance).

4. Identification criteria

$$Q_j = \sum_{j=1}^{J} \left| St_j(m,n) - St_j^M(m,n) \right| \to \min; \quad \tau_{r\varepsilon\varepsilon} \to \min,$$

where $\tau_{r\varepsilon\varepsilon}$ is the fail time of autocorrelation error function.

5. Constraint

$$M \in \overline{1, M^{\max}}; \quad N \in \overline{1, N^{\max}}.$$

The following is required to determine optimum weight coefficients α_{ij}, as well as to determine the existence of dominating typical fractal.

Formulation of Problem III

It is given:

1. Actual structures of materials

$$St_j; \quad j = \overline{1, J}.$$

2. Set of typical fractals

$$Fr_l; \quad l = \overline{1, L}.$$

3. Model of material structure

$$St_j^M \cup \left\{\frac{Fr_l}{S_{kl}}\right\} + \varepsilon_j,$$

where S_{kl} is the attracting space (area) by actual structure of the l-th typical fractal; k is the number of attracting space; and ε_j is the model error.

4. Identification criteria

$$Q_j = \sum_{k=1}^{K} S_{kl} \sum_{m,n=1}^{M,N} \left| \frac{St_j(m,n) - St_j^M(m,n)}{S_{kl}(m,n)} \right| \to \min,$$

where K is the amount of attracting spaces.

5. Constraint

$$M \in \overline{1, M^{\max}}; \quad N \in \overline{1, N^{\max}}; \quad K \in \overline{1, K^{\max}},$$

where $K^{\max}$ is the maximum amount of attracting spaces.

The following is required to determine sizes of the space S_{kl} for Fr_l, as well as to form a structure model on the basis of combination Fr_l/S_{kl}.

IDENTIFICATION BY FRACTAL MODELS GENERATED BY FORMATION MECHANISMS OF FRACTAL STRUCTURE

In the second variant of the problem's solution, the image of actual material structure is identified by the formation of fractal structure.

In the most common form, formulation of the problem for identifying material structure using generation of fractals is as follows.

It is given:

1. Actual structures of material

$$St_j; \quad j = \overline{1, J}.$$

2. Mathematical notation of fractal formation [6]:

$$X_{n+1} = f(X_n) = X_n^2 + c;$$

$$X_{n+1} = f(X_n) = (1+r)X_n - rX_n^2,$$

where r is the coefficient of increment.

3. Algorithm of fractal generation

$$Fr_l(r,c); \quad l = \overline{1, L}.$$

4. Identification criterion

$$Q_j = \sum_{m;n=1}^{M,N} \left| St_j(m,n) - Fr_l(m,n,c) \right| \to \min.$$

5. Constraint

$$M \in \overline{1, M^{\max}}; \quad N \in \overline{1, N^{\max}}; \quad L \in \overline{1, L^{\max}},$$

where $L^{\max}$ is the maximum amount of mathematical notations.

It is required to determine the parameters r, c, which minimize the variable Q.

CONCLUSIONS

Four formulations of the problem of material structure identification are given based on fractal presentations. The first three formulations of the problem of material structure identification are developed for the models based on database of known (typical) fractals. The fourth formulation is for material structure identification by generated fractal models.

During the subsequent stages on the basis of these problem formulations, the methods and algorithms of material structure identification were developed based on fractal presentations. The algorithms were written in the high-level programming language C# in Microsoft Visual Studio environment. Efficiency of the developed methods and algorithms of identification were estimated using model samples and actual data. Samples of 35KhGSA steel were used as actual data, the laboratory base was located at the Materials Science core facilities, Siberian State Industrial University.

While identifying the actual structures of 35KhGSA steel by the methods based on models of known (typical) fractals and by the methods with generation of fractal models, approximately the same evaluations were obtained. Herewith, the methods with generation of fractal models are more proffered, since it is not necessary to select prearranged fractal structures, which is also a difficult formalizable problem.

Thus, the following data was proven: efficiency of the proposed problem formulations based on field studies; methods and algorithms of identification based on presentations of dynamic chaos; as well as their possibility to apply them for the identification of actual structures of materials and rocks for mining, metallurgical industry, and materials science.

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