Journal of Physics: Conference Series

PAPER • OPEN ACCESS

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To cite this article: A V Zimin *et al* 2020 *J. Phys.: Conf. Ser.* **1691** 012155

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Use of generalized dual problem solutions in the game model of competence management problem

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Abstract. The game model of the competence management problem is discussed in this article, in which the players are the center and m of agents. The model is based on the formation by agents of their optimal strategies through changes (in accordance with their preferences) of basic solutions developed by the center. As basic solutions the center offers to agents solutions of the optimization problem of personalized competence management corresponding to optimal solutions of the generalized dual problem of network programming generated by it. These solutions are meet the best use of resources to maximize staff competencies. However, they were developed without taking into account the target settings of the center and agents. Agents themselves, or by entering into coalitions with other agents, adjust basic solutions or synthesize new ones based on them in accordance with their preferences. The center evaluates the basic and proposed by agents solutions according to its own criterion. The solution of the game is a solution developed by agents and delivering the maximum to a complex indicator of the effectiveness of solutions, formed taking into account the weights of evaluations of decisions of both the center and agents.

1. Introduction

Generalized dual problem (GDP) is formulated as a problem for finding the minimum of the upper boundary (the maximum of the lower boundary) for the optimum obtained when solving discrete linear and non-linear problems by network programming. The need to solve GDP arises when, in order to fulfill the conditions of applicability of the method, the right part of the task constraint must be divided into several unknown addends. The GDP consists in finding a such partition that delivers corresponding minimum (maximum). The iterative procedure for solving the generalized dual problem of network programming (GDP) generated by the task of personalized competence management is considered in equation [1]:

$$q = \sum_{j=1}^{m} \sum_{i=1}^{n_j} \sum_{k}^{k_j} q_{ji}^{k} x_{ji}^{k} = \sum_{j=1}^{m} q_j(x_j) \to \max , \tag{1}$$

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Journal of Physics: Conference Series **1691** (2020) 012155 doi:10.1088/1742-6596/1691/1/012155

$$c=\sum_{j=1}^{m}\sum_{i=1}^{n_j}\sum_{k}^{k_j}c_{ji}x_{ji}^{k}\le c^{*},\tag{2}$$

$$\sum_{i=1}^{n_j}\sum_{k}^{k_{ji}}x_{ji}^{k}\ge k_j^{*},\quad j=\overline{1,m}.\tag{3}$$

Here $\{\{p_{ji}\,|\,i=\overline{1,n_j}\}\,|\,j=\overline{1,m}\}$ – is a set of education programs, j – business-process number, i – training program number, p_{ji} – i-th training program for j-th business process, n_j – number of training programs for j-th process, m is the number of business processes, $c_{ji}=c(p_{ji})$ – the cost of training of one user by program p_{ji}, $q_{ji}^{k}=q_{ji}^{k}(p_{ji})$ – "increment" of competence of the k-th user as a result of program training p_{ji}, k_j –the number of users of the j-th business process, k_j^{*} – the minimum number of users of the j-th business process, which must be trained, c^{*} – the maximum amount of funds for training, x_{ji}^{k} – is a variable that is 1 if the k-th user of the j-th process is subject to program training p_{ji}, and is 0 otherwise.

Let's call as basic the solutions of the initial problem (1) – (3) corresponding to the optimal solutions of its GDP. These solutions provide the best use of resources for maximum possible increases in competencies. However, for the game setting of the task (with different, but not opposite interests of the players), these solutions cannot be used in their pure form, since they do not take into account the target preferences of the center and agents [2].

Let the set:

$$\{\{\{x_{ji}^{k}\,|\,i=\overline{1,n_j}\}\,|\,k=\overline{1,k_j}\}\,|\,j=\overline{1,m}\}^{\theta},\theta=\overline{1,\theta^{b}}\tag{4}$$

describes a set θ^{b} of basic solutions of the initial problem corresponding to the solutions of its GDP. Decisions (4) are effective according to the criterion $q=\sum_{j=1}^{m}q_j(x_j)$. Let's assume following axiom: agents can build a solution that satisfies them by modifying the solutions of the base set (4), or synthesizing new solutions satisfying them. [3–7]. Note that each of the basic solutions differs in effectiveness for agents, as the j-th agent evaluates a separate solution by its fragment $(q_j,c_j,\{\{x_{ji}^{k}\,|\,i=\overline{1,n_j}\}\,|\,k=\overline{1,k_j}\}),\ j=\overline{1,m}$. Let's assume that the center evaluates the effectiveness of the basic decision according to the criterion

$$\Delta q^{ud}=\sum_{j=1}^{m}\left|\,q_j^{ud}-\frac{q_j(x_j)}{c_j(x_j)}\right|.\tag{5}$$

2. Move sequence of players and the procedure for the game solution definition

Let build the solution of the game according to the following procedure:

1. Center:

1.1 solves the GDP problem generated by the initial task of personalized competence management (1) – (3), and generates a set (4) of basic solutions;

1.2 calculates for each basic solution the value of the criterion (5), evaluates this value in the scores of the given rank scale of measurement and communicates to the agents the list of basic solutions and their estimates of their effectiveness.

2. Agents:

2.1 adjust (if considers expedient) "their" fragments $\{\{x_{ji}^{k}\,|\,i=\overline{1,n_j}\}\,|\,k=\overline{1,k_j}\}$ of basic solutions, keeping the costs cj for these solutions unchanged (such adjustments may change effect of training);

2.2 adjust, by entering into coalitions with other agents, fragments of basic decisions, keeping the corresponding total coalition costs unchanged (such adjustments change not only the effect of training, but also budgets of agents joined the coalition);

2.3 adjust, by entering into coalitions with other agents and the center, various fragments of individual basic solutions, up to the synthesis of new solutions (by changing the basic ones);

2.4 form a set of solutions corresponding to their objectives:

$$((q_\theta, c_\theta, \{\{\{x_{ji}^k | i=\overline{1,n_j}\} | k=\overline{1,k_j}\} | j=\overline{1,m}\})^\theta, \theta=\overline{1,\theta^{ag}}). \tag{6}$$

2.5. make an evaluation $q_{jb}^\theta, j=\overline{1,m},$ of solutions (6) in points of a given rank measurement scale by corresponding fragments of decisions.

3. Center:

3.1 calculates values of criterion (5) for the solutions generated by agents and convert to scores of the scale of measurement of given rank;

3.2 calculates (based on own scoring and scoring of the agents) the value of a comprehensive performance indicator Q^θ for each solution proposed by agents:

$$Q^\theta = \alpha_c \Delta q_b^{ud\theta} + \sum_{j=1}^{m} \alpha_j q_{jb}^\theta, \theta = \overline{1,\theta^b}, \tag{7}$$

where α_c and $\alpha_j, j=\overline{1,m},$ – are weighting factors for the evaluations of center and agents:

$$\alpha_c + \sum_{j=1}^{m} \alpha_j = 1. \tag{8}$$

4. Based on scoring (7) a center determine the best solution and declares it as solution of the game.

3. Example

Let the basic solutions obtained by the center as a result of solving GDP problem generated by the initial task of personalized competence management (1) – (3) are described in table 1.

Table 1. Basic solutions of the game model of competence management.

№	1	2	3	4	5	6
q	72	72	72	72	72	72
k	15	15	14	14	15	15
c	1100	1100	1100	1100	1100	1100
x_{1i}^1	101	101	101	101	100	100
x_{1i}^2	110	110	110	110	110	110
x_{1i}^3	000	000	000	000	000	000
x_{1i}^4	100	100	100	100	100	100
x_{1i}^5	001	001	000	000	000	000
x_{1i}^6	110	110	110	110	110	110
x_{1i}^7	100	100	100	100	100	100
x_{1i}^8	100	100	100	100	100	100
x_{2i}^1	10	10	10	10	10	10
x_{2i}^2	10	10	10	10	10	10
x_{2i}^3	00	00	00	00	00	00

x_{2i}^4	10	10	10	10	10	10
x_{2i}^5	00	00	00	00	00	00
x_{2i}^6	10	00	10	00	10	00
x_{2i}^7	10	10	10	10	10	10
x_{3i}^1	01	01	11	11	11	11
x_{3i}^2	01	01	01	01	01	01
x_{3i}^3	00	00	00	00	00	00
x_{3i}^4	01	01	01	01	01	01
x_{3i}^5	00	00	00	00	10	10
x_{3i}^6	00	01	00	01	00	01

Some features of the basic solutions fragments are shown in table 2.

Table 2. Features of basic solutions.

N	1	2	3	4	5	6
q	72	72	72	72	72	72
k	15	15	14	14	15	15
c	1100	1100	1100	1100	1100	1100
q_1^θ	44	44	39	39	34	34
c_1^θ	668	668	578	578	488	488
q_2^θ	17	14	17	14	17	14
c_2^θ	270	216	270	216	270	216
q_3^θ	11	14	16	19	21	24
c_3^θ	162	216	252	306	342	396

Values of performance indicators of basic solutions calculated by the center and results of conversion of these values into 10-point scale of measurement by formula:

$$\Delta q_b^{ud\theta} = \frac{10(\Delta q_{max}^{ud} - \Delta q^{ud})}{\Delta q_{max}^{ud} - \Delta q_{min}^{ud}}, \tag{9}$$

were Δq_{max}^{ud} and Δq_{min}^{ud} – maximum and minimum values $\Delta q^{уд}$ on a variety of basic solutions, are shown in table 3.

Table 3. Scores $\Delta q^{ud\theta}$ and $\Delta q_b^{ud\theta}$ given by the center to basic solutions.

θ	1	2	3	4	5	6
$\Delta q^{ud\theta}$	0.0516	0.0504	0.0488	0.0492	0.0489	0.0500
$\Delta q_b^{ud\theta}$	0.00	4.41	10.00	8.39	9.61	5.85

Note that according to the criterion $q = \sum_{j=1}^{m} q_j(x_j)$ (table 1) all basic solutions are equally good, and according to criterion (5), which guides the center, they vary considerably. This criterion identifies the third, fifth and fourth as the best solutions.

2. Let the result of autonomous and coalition work of players on adjustment and synthesis of decisions will be a set of 4 solutions, table 4. The first two and the sixth basic solution are excluded

from consideration by agents, the third and fourth are corrected (in table 4 this is the first and second solution), the fifth basic solution remained unchanged (in the table this is the third solution). The last (fourth) solution was built by agents independently.

Table 4. Results of agents working process with basic solutions.

N	1	2	3	4
q	72	72	72	71
k	14	14	15	12
c	1100	1100	1100	1100
x_1	101100010 100000110 100100	101110000 100000110 100100	100110000 100000110 100100	101110000 100001110 100100
x_2	10100010 001010	10100010 000010	10100010 001010	00100001 000011
x_3	11010001 0000	10010101 0001	11010001 1000	10010000 0000

Table 5 shows the features of the solutions proposed by agents.

Table 5. Features of solutions produced by agents.

θ	1	2	3	4
q	72	72	72	71
k	14	14	15	12
c	1100	1100	1100	1100
q_1^{θ}	37	39	34	39
c_1^{θ}	578	578	488	578
q_2^{θ}	17	14	17	14
c_2^{θ}	270	216	270	216
q_3^{θ}	16	18	21	19
c_3^{θ}	252	306	342	306

Scores $q_{jб}^{\theta}, j=\overline{1,m},$ (in 10-points scale) of the solutions proposed by agents are given in table 6.

Table 6. Scores of agents to the developed solutions.

N	1	2	3	4
$q_{1б}^{\theta}$	7	8	6	7
$q_{2б}^{\theta}$	8	5	8	5
$q_{3б}^{\theta}$	5	7	8	7

3. Scores of a center to the proposed solutions of agents (including scores in 10-points scale of measurement) are given in table 7.

4. Let $\alpha_c = 0.40$ and $\alpha_j = 0.2, j=\overline{1,3}$. Based on ratio (7), the center (with score $Q^1 = 2.96$) announces the first solution proposed by agents as the solution of the game.

Table 7. Scores of center to proposed solutions.

N	1	2	3	4
$\Delta q^{ud\theta}$	0.0453	0.0460	0.0489	0.0457

Journal of Physics: Conference Series **1691** (2020) 012155 doi:10.1088/1742-6596/1691/1/012155

$\Delta q_b^{ud\theta}$	7.30	6.00	0.00	6.50

4. Conclusion

Use of game models in accordance with the principle of open management formulated by V.N. Burkov is preferable, since they allow better reflection of the interests of those who participate in solving the problem. With an optimization approach participants, at best, are involved in the formation of a problem formal model common to all (selection of criteria and constraints [7, 8]) and are forced to agree with both the procedure for forming an optimal solution and the result of the procedure (obtained solution).

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